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Hyperchaotic System with Unstable Oscillators

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A simple electronic system exhibiting hyperchaotic behaviour is described. The system includes two nonlinearly coupled 2nd order unstable oscillators, each composed of an LC resonance loop and an amplifier. The system is investigated by means of numerical integration of appropriate differential equations, PSPICE simulations and hardware experiments. The Lyapunov exponents are presented to confirm hyperchaotic mode of the oscillations.

Key words: hyperchaotic behaviour, coupled unstable oscillators, Lyapunov exponents.

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1 Introduction

In contrast to simple three-dimensional chaotic systems which are characterized with a single positive Lyapunov exponent (LE) the higher-dimensional hyperchaotic ones have two or more positive LEs, that is they are unstable in more than one direction. An example of a hyperchaotic system with two positive LEs has been first provided by Rössler in the form of four differential equations describing some hypothetical chemical reaction [1]. The first experimental observation of hyperchaos has been described by Matsumoto, Chua and Kobayashi [2]. They found in a 4th order electronic circuit very complicated oscillations with two positive LEs. Later a large number of hyperchaotic systems have been proposed, e.g. [3-11]. The interest in hyperchaotic systems is strongly stimulated by their possible application to information technologies, specifically to secure communications [12-14]. Very recently a hyperchaotic system including two nonlinearly coupled unstable oscillators has been suggested [15]. Experimentally each of the oscillators is implemented by means of an LC resonance circuit in parallel to a negative impedance converter [16] used for the two-terminal negative resistance device. In the present paper we describe an alternative way to implement the unstable oscillators. Amplifiers are employed as four-terminal devices and inserted in series with the LC resonance loops to introduce negative losses. Such solution provides the possibility to exploit not only low frequency operational amplifiers commonly used for building negative impedance converters, but also fast transistors to produce hyperchaos at high frequencies.

2 System description

The system is shown in Figure 1. The electronic circuit consists of two 2nd order LC based oscillators. Due to the amplifiers k1 and k2 both oscillators are unstable. The oscillators are coupled by means of a diode as a nonlinear device.

By introducing the notations

$$x = \frac{U_{C1}}{B_p}, y = \frac{\rho i_{L1}}{B_p}, z = \frac{U_{C2}}{B_p}, w = \frac{\rho i_{L2}}{B_p},$$

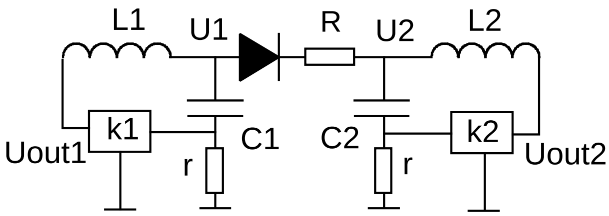

FIG. 1. Hyperchaotic oscillator.

$$\theta = \frac{t}{\tau},\ \dot{u} = \frac{du}{d\theta},\ \rho = \sqrt{\frac{L_1}{C_1}},\ \tau = \sqrt{L_1 C_1},$$
$$a = (k_1 - 1)d,\ b = (k_2 - 1)d,$$
$$c = \frac{\rho}{r_d + R + 2r},\ d = \frac{r}{\rho},\ \epsilon = \frac{C_2}{C_1},\ \mu = \frac{L_2}{L_1},$$

where B_p is the break-point voltage of the diode I-V characteristic, the following set of equations is derived

$$\begin{aligned} \dot{x} &= y - G, \\ \dot{y} &= -x + a(y - G), \\ \epsilon \dot{z} &= w + G, \\ \mu \dot{w} &= -z + b(w + G). \end{aligned} \tag{1}$$

Here $G \equiv G(x, y, z, w) = 0.5c(|x - z + d(y - w) - 1| + x - z + d(y - w) - 1)$.

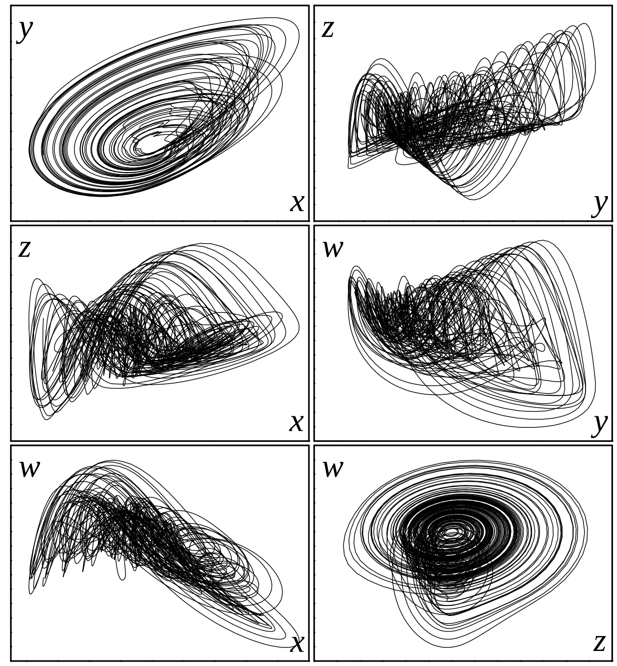

FIG. 2. Six projections of the four-dimensional hyperchaotic attractor from Eqn.(1) at a =0.6, b =0.2, c =1.5, d =0.1, ϵ =0.31, μ =0.33.

3 Numerical simulation

Typical phase portraits are shown in Figure 2. Power spectra are demonstrated in Figure 3. There are two positive LEs, confirming hyperchaotic mode of the oscillations (Figure 4).

4 PSPICE simulation

In recent years circuit simulators such as PSPICE have been successfully applied to chaotic and hyperchaotic oscillators [8,15]. Here we demonstrate the possibilities of PSPICE once again by means of simulating the circuit in Figure 1 and comparing the results with the experimental ones. The phase portrait $U_{out1} - U_{out2}$, that is composed of two experimentally accessible signals is presented in Figure 5.

5 Experimental

The hyperchaotic system in Figure 1 has been built experimentally using the following element values L_1 =256mH, L_2 =84mH, C_1 =220nF, C_2 =68nF, R = 510Ω, r = 110Ω. The base frequencies are $f_1 = (2\pi\sqrt{L_1 C_1})^{-1}$ ≈670Hz and $f_2 = (2\pi\sqrt{L_2 C_2})^{-1}$ ≈2100Hz, the characteristic resistances of both LC loop $\rho \approx 1.1k\Omega$. The diode is the 1N914 type general purpose signal diode, the amplifiers are built using the LM741 opamps. The experimental phase portrait is presented in Figure 6. One can see good agreement between the PSPICE and experimental results.

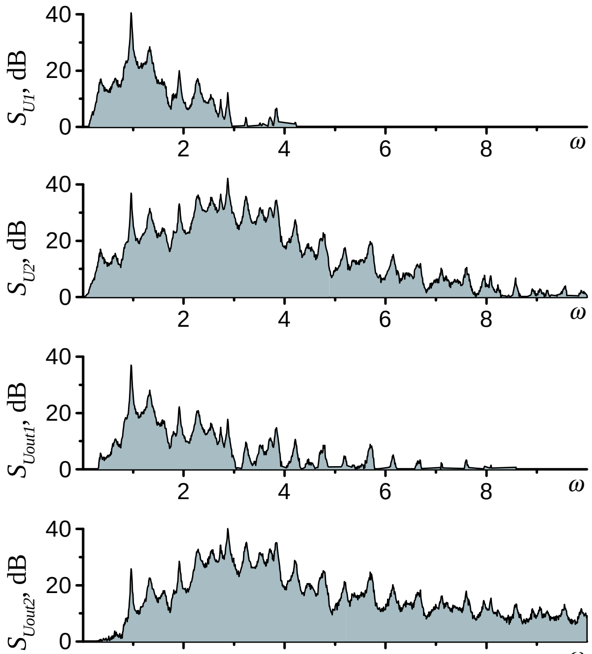

FIG. 3. Power spectra from Eqn.(1). Parameters are the same as in Figure 2. $U_1 \propto x+d(y-G)$, $U_2 \propto z+d(w+G)$, $U_{out1} \propto k_1 d(y-G)$, $U_{out2} \propto k_2 d(w+G)$.

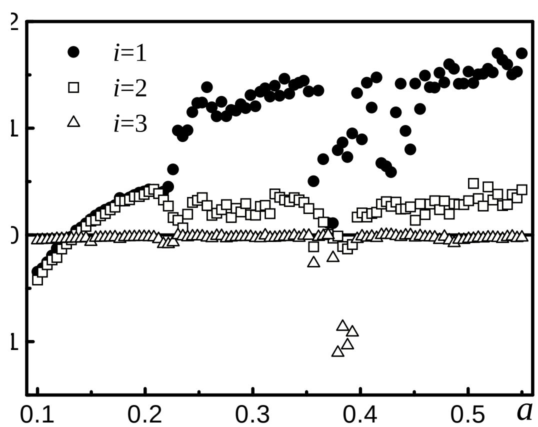

FIG. 4. Three largest Lyapunov exponents against parameter a from Eqn.(1). Other parameters are in Figure 2.

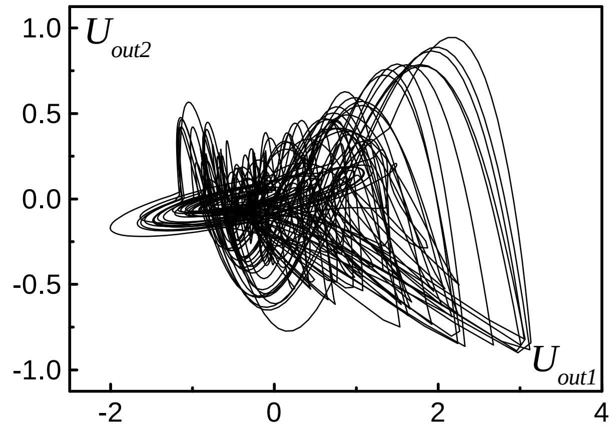

FIG. 5. PSPICE phase portrait U_{out1} versus U_{out2}. Circuit element values are given in Section 5, $k_1 = 7, k_2 = 3$. The numbers along the axes are in volts.

6 Conclusions

We have designed and investigated a hyperchaotic system composed of two nonlinearly coupled unstable linear oscillators. Oscillations are characterized with two positive Lyapunov exponents. Due to the four-terminal amplifier used for the negative resistors the system is expected to operate at high frequencies as well provided fast transistors are employed.

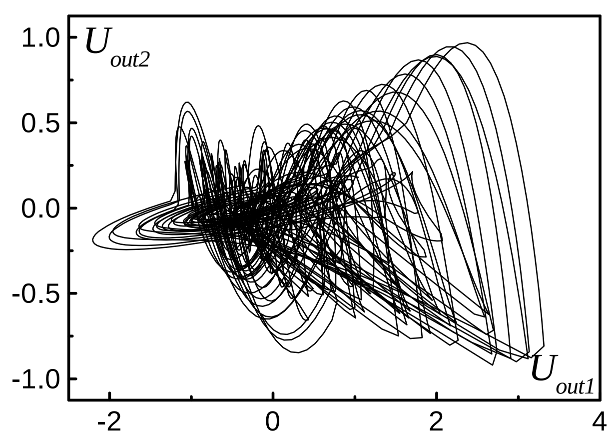

FIG. 6. Experimental phase portrait U_{out1} versus U_{out2}, $k_1 = 7, k_2 = 3$. The numbers along the axes are in volts.

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